Lecture 7: Linear models

Firas Moosvi (Slides adapted from Varada Kolhatkar)

Announcements

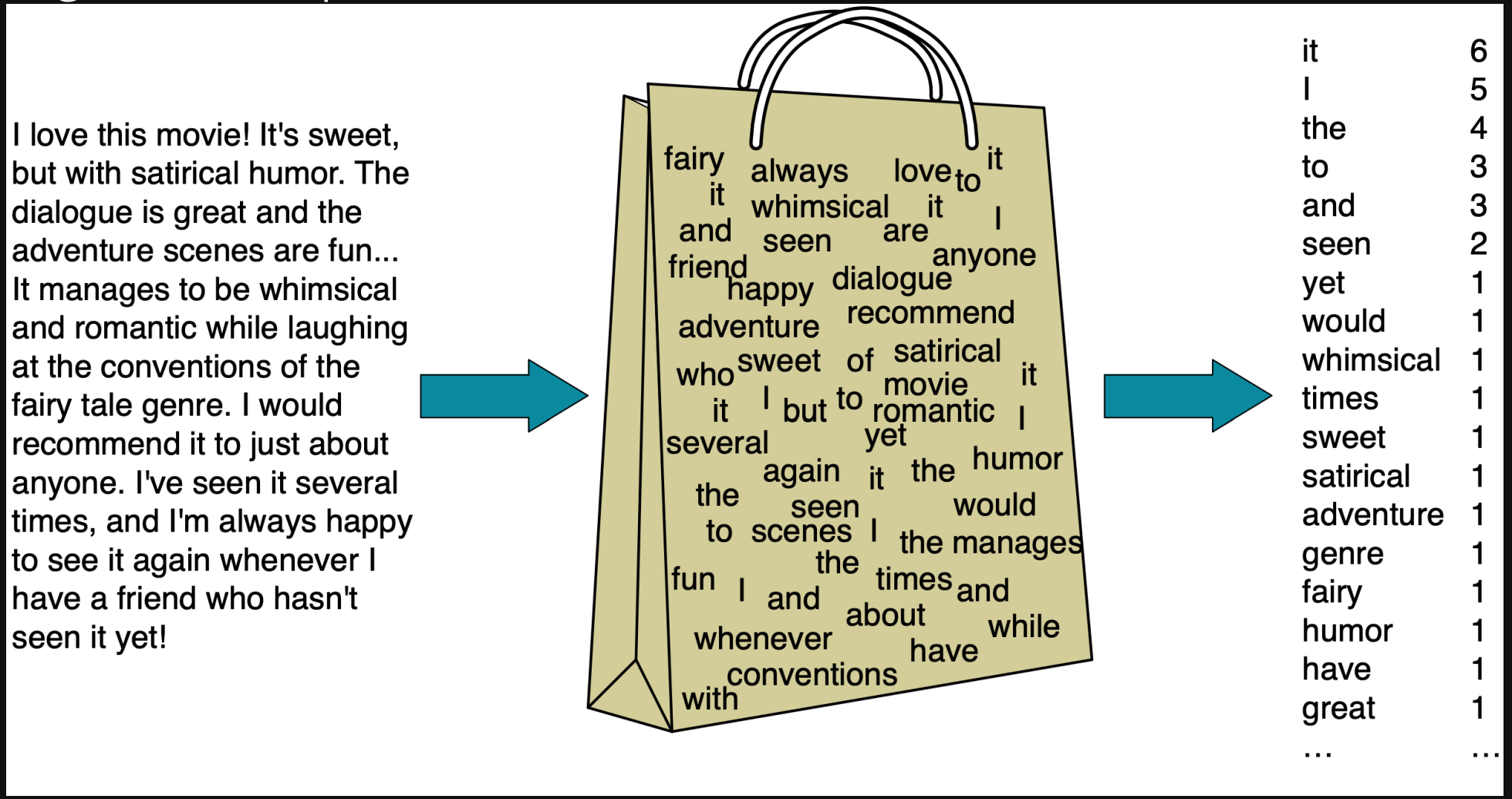
- CPSC 330 Office Hours are now posted on Ed Discussion
- Reminder: Test 1 window is ongoing, so:
  - forums will be closed,
  - OH are cancelled,
  - Tutorials are still running, but do not cover testable content
- Test 2 & 3 reservations are open!

Announcements (cont'd)

- Test viewing windows are also open
  - You will need to book a viewing session to review your Tests
  - This is your opportunity to learn from your mistakes and look at the correct answers for the Tests
  - You may also request regrades at this time
- HW 3 and 4 are released!
  - You may work on these in pairs

Recap: Dealing with text features

- Preprocessing text to fit into machine learning models using text vectorization.
- Bag of words representation



Recap: `sklearn CountVectorizer`

- Use `scikit-learn`'s `CountVectorizer` to encode text data
- `CountVectorizer`: Transforms text into a matrix of token counts
- Important parameters:
  - `max_features`: Control the number of features used in the model
  - `max_df`, `min_df`: Control document frequency thresholds
  - `ngram_range`: Defines the range of n-grams to be extracted
  - `stop_words`: Enables the removal of common words that are typically uninformative in most applications, such as "and", "the", etc.

(iClicker) Exercise 6.2

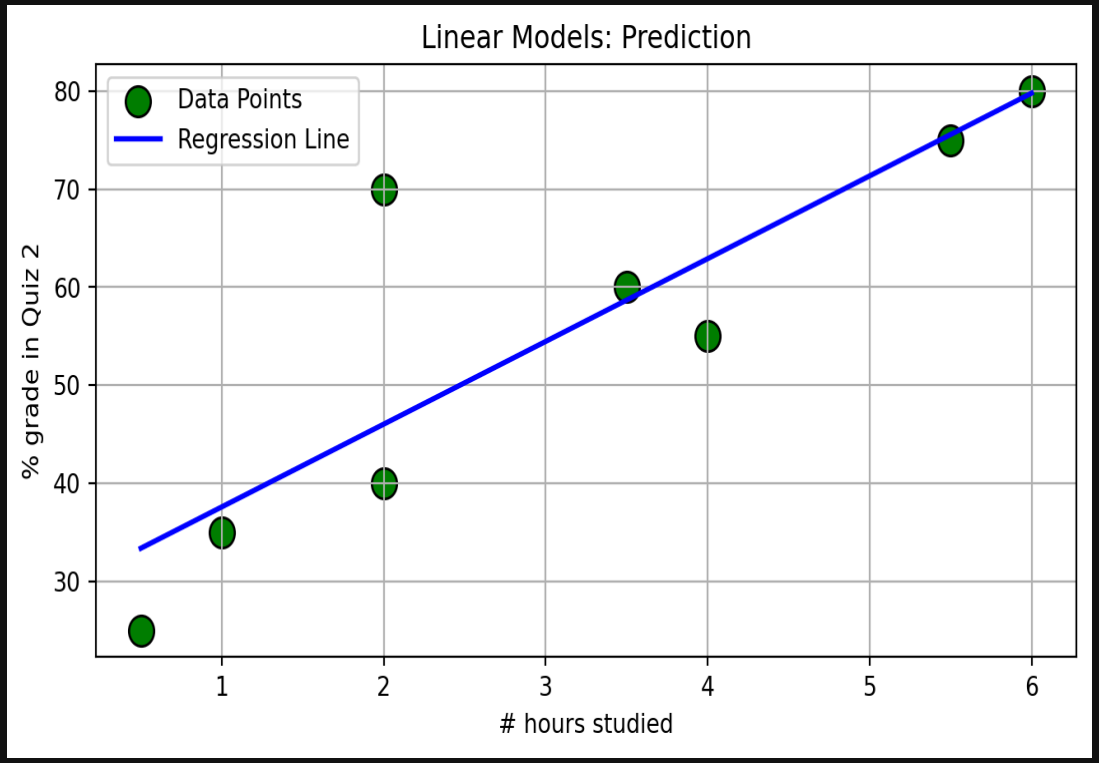
Select all of the following statements which are TRUE.

- a. `handle_unknown="ignore"` would treat all unknown categories equally.
- b. As you increase the value for `max_features` hyperparameter of `CountVectorizer` the training score is likely to go up.
- c. Suppose you are encoding text data using `CountVectorizer`. If you encounter a word in the validation or the test split that's not available in the training data, we'll get an error.
- d. In the code below, inside `cross_validate`, each fold might have slightly different number of features (columns) in the fold.

```
1 pipe = (CountVectorizer(), SVC())
2 cross_validate(pipe, X_train, y_train)
```

Linear models

- Linear models make an assumption that the relationship between X and y is linear.
- In this case, with only one feature, our model is a straight line.
- What do we need to represent a line?
  - Slope (w_1): Determines the angle of the line.
  - Y-intercept (w_0): Where the line crosses the y-axis.



- Making predictions:
  $y_{hat} = w_1 \times \# \text{ hours studied} + w_0$

`Ridge` vs. `LinearRegression`

- Ordinary linear regression is sensitive to **multicolinearity** and overfitting
- Multicolinearity: Overlapping and redundant features. Most of the real-world datasets have colinear features.
- Linear regression may produce large and unstable coefficients in such cases.
- `Ridge` adds a parameter to control the complexity of a model. Finds a line that balances fit and prevents overly large coefficients.

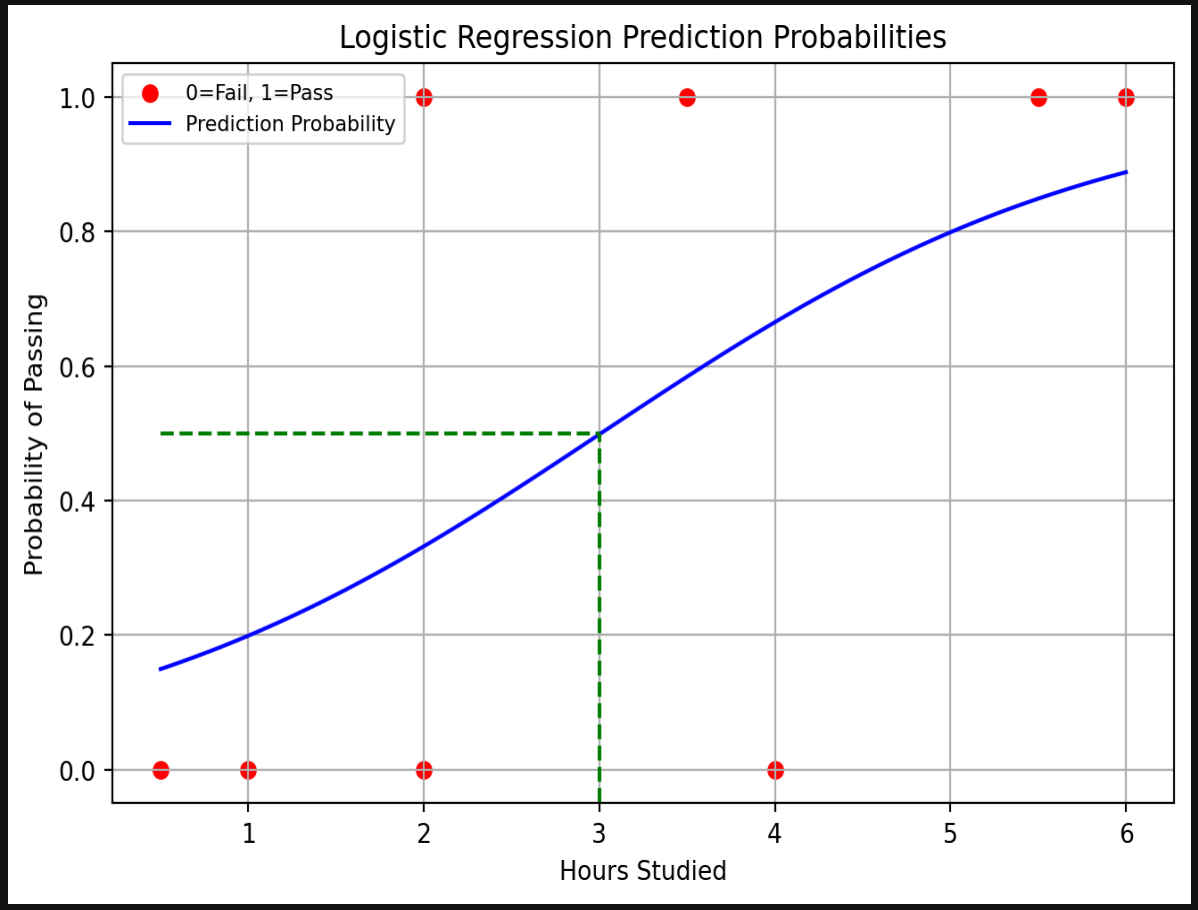
When to use what?

- `LinearRegression`
  - When interpretability is key, and no multicollinearity exists
- `Ridge`
  - When you have **multicollinearity** (highly correlated features).
  - When you want to prevent **overfitting** in linear models.
- **In this course, we'll use `Ridge`.**

Logistic regression

- Suppose your target is binary: pass or fail
- Logistic regression is used for such binary classification tasks.
- Logistic regression predicts a probability that the given example belongs to a particular class.
- It uses **Sigmoid function** to map any real-valued input into a value between 0 and 1, representing the probability of a specific outcome.
- A threshold (usually 0.5) is applied to the predicted probability to decide the final class label.

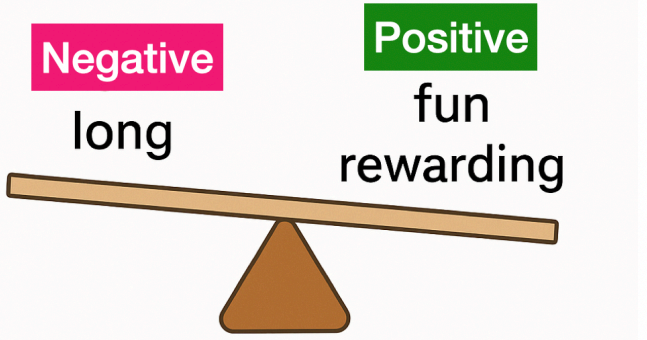
Logistic regression: Decision boundary



- Sigmoid Function:
  $\hat{y} = \sigma(w^\top x_i + b) = \frac{1}{1+e^{-(w^\top x_i+b)}}$
- The decision boundary is the point on the x-axis where the corresponding predicted probability on the y-axis is 0.5.

Sentiment analysis example

The lectures are long but the demos are fun and the assignments are rewarding.



	Coefficient
excellent	0.79
amazing	0.62
fun	0.61
rewarding	0.54
...	...
long	-0.68
boring	-0.73
awful	-0.87
worst	-1.13

$$\hat{y} = \sigma(1 \times 0.61 + 1 \times 0.54 + \ldots + 1 \times (-0.68))$$

- Logistic regression learns **coefficients for each word** from training data.
- Positive coefficients ⟶ push prediction toward positive class.
- Negative coefficients ⟶ push prediction toward negative class.
- In this example, positive words (*fun*, *rewarding*) outweigh the negative word (*long*), so the overall sentiment is likely **positive**.

Parametric vs. non-Parametric models (high-level)

- Imagine you are training a logistic regression model. For each of the following scenarios, identify how many parameters (weights and biases) will be learned.
- Scenario 1: 100 features and 1,000 examples
- Scenario 2: 100 features and 1 million examples

Parametric vs. non-Parametric models (high-level)

Parametric

- Examples: Logistic regression, linear regression, linear SVM
- Models with a fixed number of parameters, regardless of the dataset size
- Simple, computationally efficient, less prone to overfitting
- Less flexible, may not capture complex relationships

Non parametric

- Examples: KNN, SVM RBF, Decision tree with no specific depth specified
- Models where the number of parameters grows with the dataset size. They do not assume a fixed form for the functions being learned.
- Flexible, can adapt to complex patterns
- Computationally expensive, risk of overfitting with noisy data

(iClicker) Exercise 7.1

Select all of the following statements which are TRUE.

- a. Increasing the hyperparameter `alpha` of `Ridge` is likely to decrease model complexity.
- b. `Ridge` can be used with datasets that have multiple features.
- c. With `Ridge`, we learn one coefficient per training example.
- d. If you train a linear regression model on a 2-dimensional problem (2 features), the model will learn 3 parameters: one for each feature and one for the bias term.

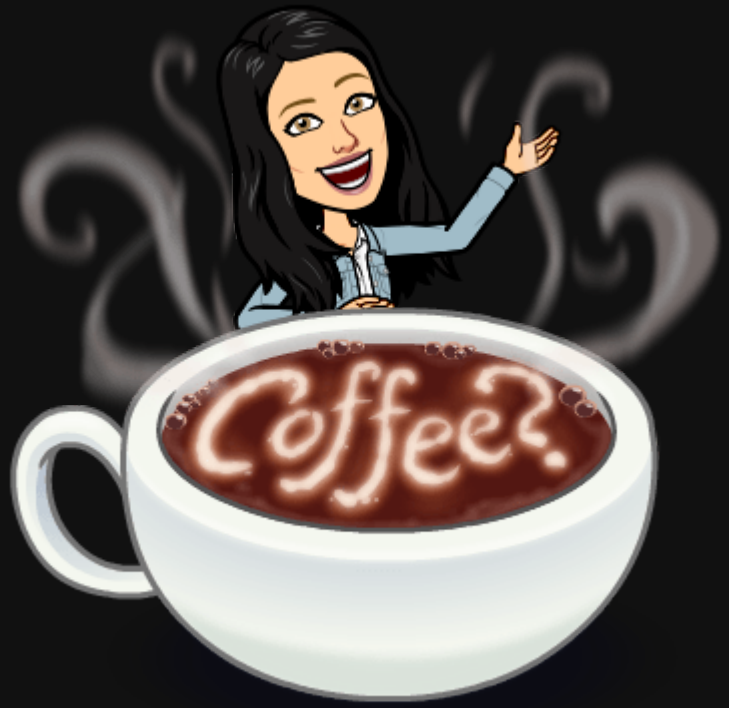
(iClicker) Exercise 7.2

Select all of the following statements which are TRUE.

- a. Increasing logistic regression's `C` hyperparameter increases model complexity.
- b. The raw output score can be used to calculate the probability score for a given prediction.
- c. For linear classifier trained on d features, the decision boundary is a $d-1$-dimensional hyperparlane.
- d. A linear model is likely to be uncertain about the data points close to the decision boundary.

Break

Let’s take a break!



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(Optional) Multinomial logistic regression

Softmax Function for Probabilities

Given an input, the probability that it belongs to class $j \in \{1, 2, \ldots, K\}$ is calculated using the **softmax function**:

$$P(y = j \mid x_i) = \frac{e^{w_j^\top x_i + b_j}}{\sum_{k=1}^{K} e^{w_k^\top x_i + b_k}}$$

- x_i is the i^{th} example
- w_j is the weight vector for class j.
- b_j is the bias term for class j.
- K is the total number of classes.

Making Predictions

1. **Compute Probabilities**:
   For each class j, compute the probability $P(y = j \mid x_i)$ using the softmax function.
2. **Select the Class with the Highest Probability**:
   The predicted class $\hat{y}$ is:
   $\hat{y} = \arg\max_{j \in \{1,\ldots,K\}} P(y = j \mid x_i)$

Binary vs multinomial logistic regression

Aspect	Binary Logistic Regression	Multinomial Logistic Regression
Target variable	2 classes (binary)	More than 2 classes (multi-class)
Getting probabilities	Sigmoid	Softmax
parameters	d weights, one per feature and the bias term	d weights and a bias term per class
Output	Single probability	Probability distribution over classes
Use case	Binary classification (e.g., spam detection)	Multi-class classification (e.g., flower species)

Coming up

A few big questions remain:

- How do we tune hyperparameters?
- How do we choose our features? (feature selection, dimensionality reduction, regularization etc.)
- How do we come up with new useful features (feature engineering)
- How do we choose between different models? (different evaluation metrics, what happens if we're not happy with our test error?)
- How to deal with class imbalance?
- What do we do if we do not have targets (unsupervised learning)
- How to build models for more interesting data such as images, user preferences, or sequential data?)

Group Work: Class Demo & Live Coding

For this demo, each student should click this link to create a new repo in their accounts, then clone that repo locally to follow along with the demo from today.